[[1]](#footnote-1)

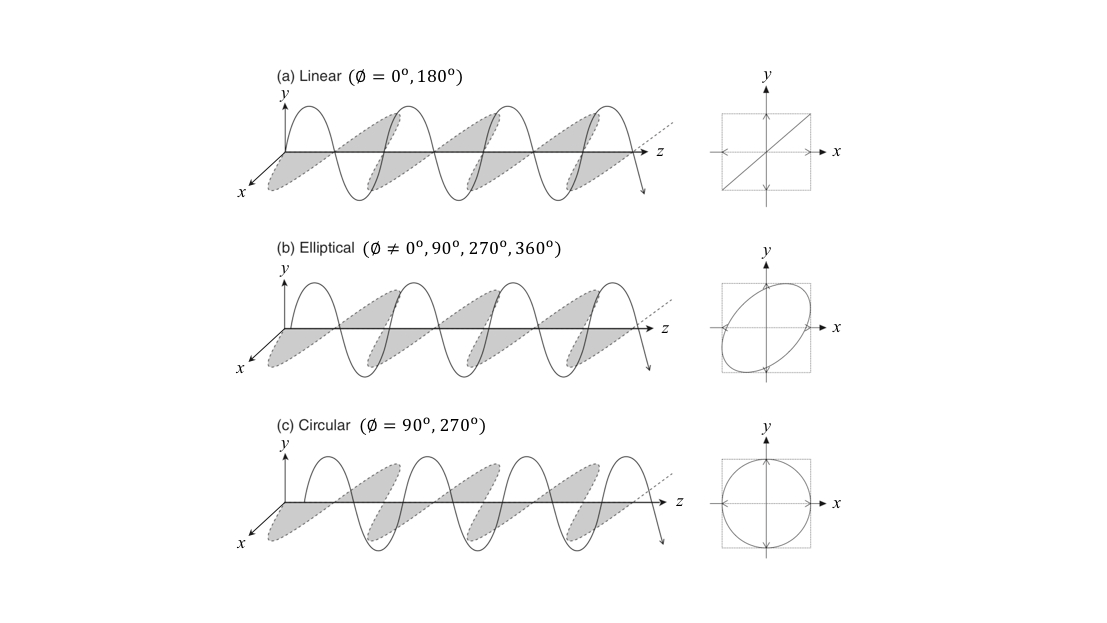


Figure 1: Illustrations of linearly, circularly and elliptically polarised light. The type of polarisation is determined by the phase difference φ between the x and y components of the electric field. Diagrams on the left show the relative positions of the x and y components of the electric field, and those on the right show the superposition of both components as viewed along the z axis. Image adapted from [4].

**Monte Carlo Naught and Crosses**

Shawn C. Y. Tan, Min Lin

*Undergraduate, Physics Department, Imperial College, London SW7 2BB*

Verification of Malus’ Law was conducted by directing linearly polarised light through a linear polariser in a polariser-analyser system. Analyses was conducted on the variation in intensity of the transmitted wave as the transmission axis of the linear polariser is rotated. The expected cosine squared relationship detailed by Malus’ Law is obtained with a coefficient of determination *R2* = 0.9991.

# Introduction

Applications of Monte Carlo methods

Naught and Crosses as a game to analyse which is similar to other games.

# Theory

## Monte Carlo Methods

Light can be classically considered as the propagation of transverse electromagnetic vector waves. For an axis of propagation along the positive *z*-axis, the electric field vector ***E***, a two-dimensional space and time varying vector, can be described as:

, (1)

Where ***E***0 is the amplitude of ***E***, ω the angular frequency, *t* the time coordinate, *k* the wave vector and *z* the position coordinate. By considering the components of ***E*** along two orthogonal axes *x* and *y* perpendicular to *z*, as illustrated in Fig. 1, ***E*** can be rewritten (at *z* = 0):

, (2)

Polarisation of light refers to the nature of the electric field of an electromagnetic wave. Specifically, the state of polarisation depends on the value of the phase difference *φ* between ***E*0,*x***and ***E*0,*y*** (shown in Fig. 1) [7]. The magnetic field is conveniently omitted as it is often perpendicular and in phase to ***E***, and knowledge of one will yield the other.

Natural light, from sources such as a blackbody, is naturally unpolarised, with *φ* varying randomly in time and hence lacking a constant state of polarisation [8]. To produce polarised light from such sources, polarisers, a type of optical filter, may be used. There exist a range of polarisers with differing optical properties. Relevant to this experiment are linear polarisers that allow only the components of ***E*** parallel to the transmission axis of the polariser through.

## Naughts and Crosses

The behaviour of polarised light when passed through a linear polariser is described by Malus’ Law:

, (3)

where *I* is the intensity of transmitted wave, *I0* the intensity of incident wave and *θ* the angle between the electric field vector and transmission axis of the polariser [9].

A polariser-analyser system is used to analyse the effect of varying *θ* on *I*. A linear trend is expected for the relationship between *I* and , with the gradient being *I*0. By measuring multiple values of *I* and *θ*, the veracity of (3) can be investigated.

# Method

## Win Condition Checking

Sub-grids

Image processing

Vectorisation

## Measuring Speed of Algorithms

## Determination of Edge

## Determination of Game Length

The set-up shown in Fig. 2 was first constructed atop an optical bench. Monochromatic red light from a LED source was used to avoid the refractive effects observed with light comprising a range of wavelengths. This was focused by the lens (focal length *f* = 160mm) onto the photodiode at a suitable magnification *m* = -0.25.

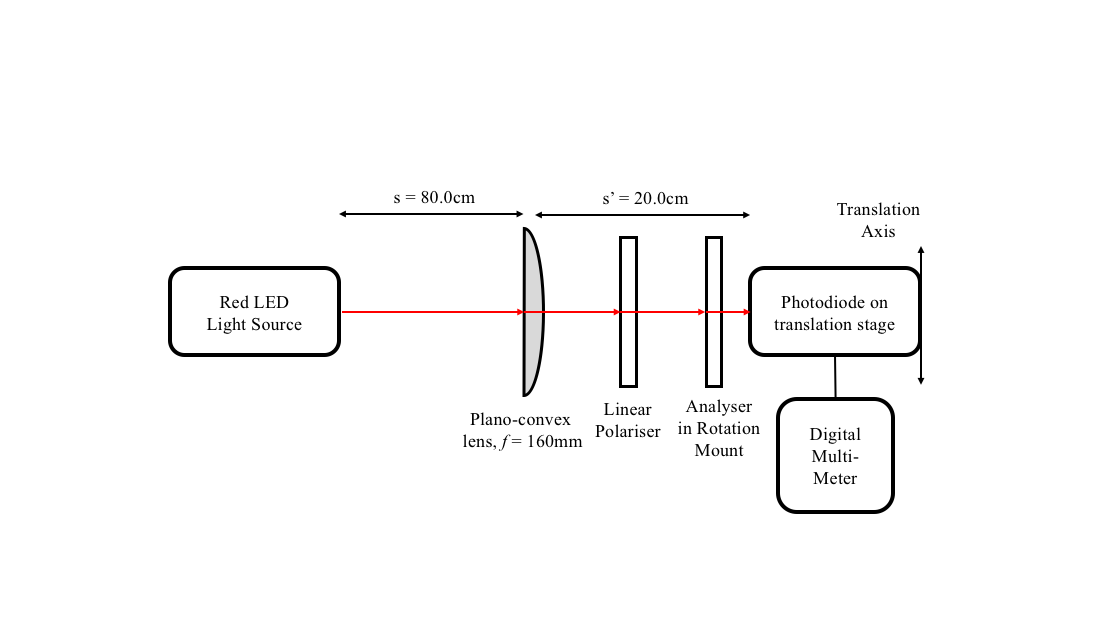


Figure 2: Polariser-analyser setup used in the experiment. The setup was placed on an optical bench. Image adapted from [8].

The object distance *s* and image distance *s’* were determined using the definition of linear magnification:

,

(4)

and the thin lens formula:

. (5)

The linear polariser served to generate a beam of linearly polarised light from the unpolarised red light. The analyser, another linear polariser mounted on a rotation mount, could be rotated along the axis parallel to the light beam such that its transmission axis forms an angle *θ* between 0o and 360o relative to that of the linear polariser.

The photodiode generates a potential difference *V* proportional to the intensity *I* of the visible light incident on it, and the connected digital multi-meter allows for the reading of the potential difference. In this way, the relative intensity of the light incident on the photodiode can be measured and quantified. The position of the photodiode was adjusted using the translation stage such that the entirety of the light beam was incident on the receiver of the photodiode. Measurements for the intensity of transmitted light were taken at intervals of 10o at a constant source intensity.

# Results, Errors and Discussion

## Speed of Algorithms

## Variation of Edge

## Variation of Game Length

The measured values of *V* (reduced by 0.004V to account for *Vback*, the voltage reading due to background light) for each corresponding value of *θ* are as shown in Table I.

Table I

Intensity of Transmitted Light at Varying *θ*

|  |  |  |  |
| --- | --- | --- | --- |
| *θ* (o) | *V* (mV) | *θ* (o) | *V* (mV) |
| 0.0 | 501 | 180.0 | 516 |
| 10.0 | 579 | 190.0 | 598 |
| 20.0 | 627 | 200.0 | 608 |
| 30.0 | 637 | 210.0 | 647 |
| 40.0 | 609 | 220.0 | 613 |
| 50.0 | 547 | 230.0 | 543 |
| 60.0 | 459 | 240.0 | 446 |
| 70.0 | 352 | 250.0 | 334 |
| 80.0 | 241 | 260.0 | 226 |
| 90.0 | 136 | 270.0 | 128 |
| 100.0 | 57 | 280.0 | 52 |
| 110.0 | 8 | 290.0 | 6 |
| 120.0 | -1 | 300.0 | 0 |
| 130.0 | 30 | 310.0 | 29 |
| 140.0 | 93 | 320.0 | 90 |
| 150.0 | 188 | 330.0 | 181 |
| 160.0 | 299 | 340.0 | 284 |
| 170.0 | 416 | 350.0 | 400 |

Table I: The values of V recorded as θ was varied from 0.0o to 350.0o.

A random error of mV was observed as fluctuations in the value of *V* on the digital multi-meter. This can be attributed to the random fluctuations in the background light and any interference with the electrical components of the photodiode. An error of 0.5o was recorded for the value of *θ* which stemmed from the smallest division on the rotation mount being 1o.

*V* was plotted against *θ*, as shown in Fig. 3. To verify Malus Law (3), the following function (with *V* and *θ* as the dependent and independent variables respectively) was fitted on the data plot using the relevant methods in Python:

, (6)

where *V*0 is the amplitude and δ is an offset based on the angle in which the linear polariser was mounted. It was observed from the data plot that the data agreed with the relationship described by Malus’ Law, but with a phase shift that necessitated the introduction of δ. *V*0 and δ were parameters that were optimised in the fit, and suitable guess values were estimated by observing the features of the data plot.

The values of *V* were then plotted against in Fig. 4, where the value of δ used was that determined by the prior fitting of function (6). The plot indicated a linear relationship between the two variables. A line of linear regression was subsequently fitted over the data points, and a strong linear fit was observed with a coefficient of determination *R* = 0.9991 obtained. This is indicative that Malus’ Law holds for the case of linear polarisers.

A noteworthy consideration is the non-zero reflection coefficient of the polariser and analyser characterised by the Fresnel equations [10]. A proportion of light incident on the polariser and analyser would experience reflection due to the change in refractive index. For light at normal incidence, the Fresnel equations can be expressed:

, (7)

## where R is the reflection coefficient, and nt and ni are the optical indices of the transmission and incident media respectively [11]. The value of R corresponding to

, (8)

## an estimated value approximately the optical index of glass, is 0.04. Considering the presence of two boundaries in both polariser and analyser, the overall reflection coefficient for the polariser-analyser setup is estimated to be 0.15. However, this has no impact on the linear regression line in Fig.4 as R can be adjusted for by multiplying the amplitude term V0 in (6) by R.

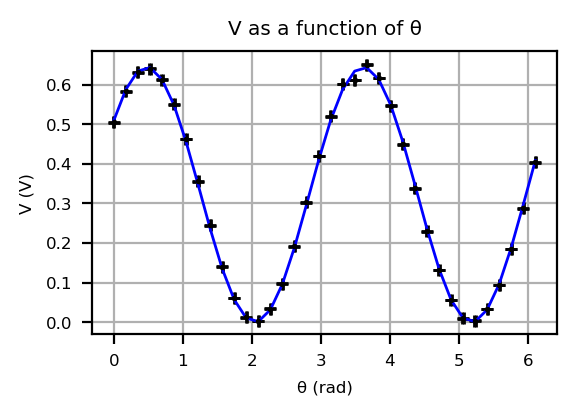


Figure 3: Plot of V, which is proportional to the intensity of light incident on the photodiode, against the angle θ between **E** and the transmission axis of the analyser. The function (6), represented by the blue line, was fitted over the data with the amplitude and phase of the function as fit parameters.

# Conclusion

The verification of Malus’ Law was conducted using a polariser-analyser set up. The variation in the intensity of polarised light transmitted through an analyser was measured as the orientation of the polariser varied. By conducting linear regression analysis, a fit with a coefficient of determination *R* = 0.9991 was obtained, which is indicative of Malus’ Law holding for polarised light incident on linear polarisers. There were no major sources of error that had direct impact on the verification of Malus’ Law. The verification can be conducted for more general with the use of light in an alternative state of polarisation or using polarisers of more complex optical properties.

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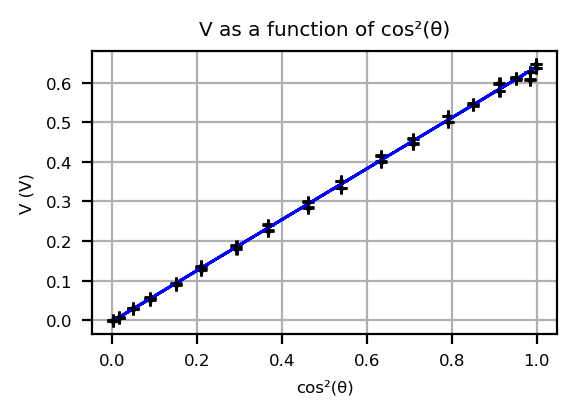


Figure 4: Plot of V against . The linear correlation between the variables was studied by fitting a line of linear regression (represented by the blue line). The line is observed to fit well with the data points, and the coefficient of determination was calculated to be R = 0.9991.

|  |  |
| --- | --- |
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1. [↑](#footnote-ref-1)